# Measurement of Small Dielectric Losses in Material with a Large Dielectric Constant at Microwave Frequencies*

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*Summary*—**A method is described for measuring dielectric losses at microwave frequencies in materials with a large dielectric constant. By observing a dielectric resonance in a sufficiently large sample, the loss tangent of the material can be obtained. Results on $SrTiO_3$ single crystals at 20 kMc are presented.**

## Introduction

Accurate measurement of small dielectric losses in a material with a large dielectric constant is difficult at microwave frequencies. If a cavity perturbation method[1] is used, it is hard to strike a balance in sample size so that the frequency shift of the cavity caused by the large dielectric constant is not too large, while the losses are large enough to be readily measurable. For a metallic cavity filled with the dielectric material, the skin losses may obscure the dielectric losses. Coaxial measurements are not free from skin losses and it also becomes more and more difficult with increasing frequency to avoid losses associated with the generation of higher-order modes.[2] An appreciable reduction of skin losses has been achieved with a dielectric cavity proposed by Hakki and Coleman,[3] but here the coupling to the cavity, the identification of the modes, and the mechanical requirements complicate the use of this method.

We propose a method which uses a sample large enough to support a dielectric resonance. The shape of the sample is not critical for the measurement of the microwave losses, and the physical size can be small compared to the dimensions of the waveguide in which it is suspended. Thus the dielectric sample, which is the resonant structure, is entirely surrounded by the exciting electromagnetic wave. It will be shown that the observed losses represent the $Q$ of the dielectric material itself. This method has been applied successfully to measure the loss tangent of $SrTiO_3$ between 3 kMc and 37 kMc in a temperature range from $-180°C$ to $250°C$.

## Theory

Strontium titanate has a relative dielectric constant of about 300 at room temperature and 1500 at $-180°C$. This large dielectric constant presents an appreciable discontinuity for the propagation of electromagnetic waves. Since the characteristic feature of a resonant structure is the existence of discontinuities, an almost arbitrary piece of $SrTiO_3$ can be used as a resonant structure provided that at least two dimensions are greater than $\lambda_0/\sqrt{\epsilon}$. This makes it possible to work with small samples. A $SrTiO_3$ sample at room temperature with dimensions the order of one millimeter is large enough to resonate at 20 kMc, since the wavelength in the sample is only about 0.9 mm.

In this section of the paper the problem of dielectric resonances in general will be considered; in the Appendix the theory for the particular case of a spherically shaped dielectric resonator will be developed in more detail, and it will be shown that the approximations made here are valid for a sphere in particular.

Let us consider a finite piece of material with a relative dielectric constant $\epsilon$ and loss tangent $\tan\delta$ in free space. For a dielectric resonance $Q_0$ will be defined as

$$Q_0 = \frac{2\pi \times \text{stored energy}}{\text{energy loss per cycle}} = \frac{\omega U}{W}, \quad (1)$$

$$U = \frac{\epsilon\epsilon_0}{2}\int_{\text{inside}} |E|^2 d\tau + \frac{\epsilon_0}{2}\int_{\text{outside}} |E|^2 d\tau, \quad (2)$$

where the first integral is taken over the electric field inside the dielectric sample and the second integral over the electric field associated with the dielectric resonance outside the sample. $\omega$ is the characteristic frequency of the dielectric resonance.

The dielectric loss will be given by

$$W = \frac{\omega\epsilon\epsilon_0 \tan\delta}{2}\int_{\text{inside}} |E|^2 d\tau, \quad (3)$$

where the integral is taken over the electric field inside the dielectric sample. With $\epsilon \gg 1$ and a rapidly decaying electromagnetic field outside the sample, the second term of the stored energy, $U$, becomes negligible compared to the energy stored inside the sample. With these

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[1] See for example, W. Von Aulock and J. H. Rowen, "Measurements of dielectric and magnetic properties of ferromagnetic materials at microwave frequencies," *Bell Sys. Tech. J.*, vol. 36, pp. 427–448; March, 1957.

[2] See W. S. Gemulla, "Measuring Microwave Properties of Ferroelectrics," Sylvania Electric Products, Inc., Mountain View, Calif., Tech. Memo. No. EDL-M246, pp. 2–11; February 10, 1960, for a review of dielectric constant and loss tangent measurements as applied to ferroelectrics.

[3] B. W. Hakki and P. D. Coleman, "A dielectric resonator method of measuring inductive capacities in the millimeter range," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 402–410; July, 1960.

approximations,

$$Q_0 = \frac{1}{\tan \delta}. \tag{4}$$

In practice the sample is placed inside a waveguide; therefore, skin losses can also contribute to losses of the dielectric resonance, but because of the rapid decay of the electromagnetic fields outside the sample, this source of loss may be neglected.

It is possible to measure the cavity $Q$ by the usual frequency variation methods,[4] but because of the special problems encountered with $SrTiO_3$, a different method was developed. $SrTiO_3$ is a paraelectric whose dielectric constant obeys a Curie-Weiss law.

$$\epsilon = \frac{C}{T - T_c} \tag{5}$$

where $C \approx 8.5 \times 10^4$ is the Curie constant and $T_c = -240°C$ is the Curie temperature. For a lossless resonant structure the frequency varies inversely with the square root of the dielectric constant, so that for $\omega_0 \gg \Delta\omega$,

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{2} \frac{\Delta T}{T - T_c},$$

where $\Delta T$ is the width of a resonant peak in the temperature scale. The loaded $Q$, $Q_L$, is related to the unloaded $Q_0$ by the coupling constant $\beta$:

$$Q_0 = (1 + \beta) Q_L.$$

Therefore,

$$\tan \delta = \frac{1}{2} \frac{\Delta T}{(T - T_c)(1 + \beta)}. \tag{6}$$

## Measurement

Fig. 1 shows the experimental set up used to measure the losses of $SrTiO_3$ at 20 kMc. At this frequency the samples were either rough cubes or spheres with dimensions the order of one millimeter. The sample was supported on a piece of polyfoam for measurements below room temperature and on fiberglass for measurements above room temperature (Fig. 2). Even though the sample was partially surrounded by polyfoam or fiber glass, convection heating kept the sample close to the temperature of the waveguide. The temperature of a resonance as measured by the thermocouple with slowly increasing and with slowly decreasing temperature, varied by less than two degrees. The sample temperature differs, therefore, at most by one degree from the thermocouple reading. From (6) this will contribute about 2 per cent to the error at $-200°C$ and much less at higher temperatures.

[4] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y.; 1957.

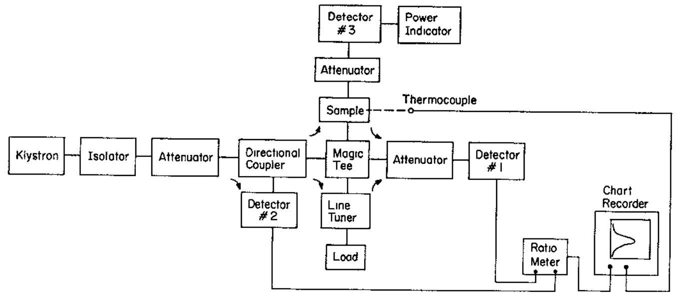


Fig. 1—Block diagram for measurement of the loss tangent at $K$ band.

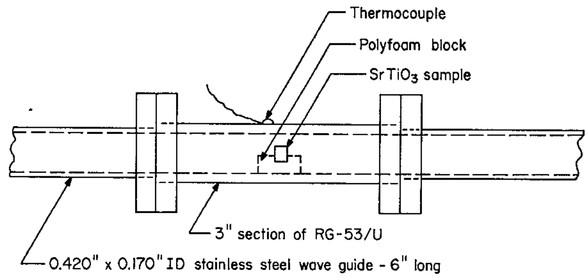


Fig. 2—Sample holder.

The sample and attenuator (which was set at 15- to 20-db attenuation to appear as a load) were placed in one arm of a hybrid tee, and a line tuner and a load were placed in the opposite arm. The line tuner was adjusted to cancel any reflections from the sample arm when the sample was not resonant. For measurements below room temperature the section of waveguide with the sample was cooled to liquid nitrogen temperature and then allowed to warm slowly. Above room temperature a coil heater was used to heat the sample. A thermocouple was soldered to the waveguide wall next to the sample and used to drive the $x$ axis of an $x$-$y$ chart recorder. The relative signal at detector 1 (Fig. 1) was plotted as a function of temperature on the chart recorder. From this plot of reflection vs temperature, $\Delta T$ and $T$ were obtained. By observing the power received at detector 3, the coupling coefficient can be determined.[5] In particular, if we assume the sample behaves as a reaction cavity, it can be shown that

$$(1 + \beta) = \sqrt{\frac{P_0}{P_{min}}},$$

where $P_0$ is the power transmitted off resonance and $P_{min}$ is the minimum power transmitted as the sample goes through resonance.

Measurements have also been made at various frequencies between 3 kMc and 37 kMc using similar methods.

[5] C. G. Montgomery, "Technique of Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y.; 1947.

## Results and Discussion

Measurements have been made on various rough cubes and spheres. Fig. 3 shows measurements made on two different samples of $SrTiO_3$. Because of the large variation of the dielectric constant in the temperature range of $-180°C$ to $250°C$, there are many temperatures at which the samples were resonant. From these resonances the loss tangent can be determined as a function of temperature for a constant frequency.

When the relative orientation of the sample in the waveguide was changed, different resonances were excited, but the same value of the loss tangent was observed. The measured value of the loss tangent was also independent of the surface finish and the size of the sample.

The main problem encountered with the cube was that resonances were partially superimposed. Because of the symmetry of a cube a large number of resonances are degenerate, but since the samples were not quite perfect this degeneracy is lifted and some peaks are split. Care must be taken to insure that there is only one peak. This problem can be reduced by using a parallelepiped with three (3) unequal sides, or spheres.

The microwave power incident on the sample must be kept at a low level to reduce any heating effects. The linewidths are only about 0.2°C near liquid nitrogen temperature. Slight heating can alter the line shape or width. In all measurements the power was reduced until the line shape and width were independent of the power level.

Obviously, this or a similar method of measuring the loss tangent can be extended to any frequency where $SrTiO_3$ or any other material with a large dielectric constant can be placed inside a section of waveguide.

## Appendix

The problem of a dielectric resonator of anything but the simplest shape is quite complicated. Because of its symmetry the sphere is the easiest shape to solve exactly.[6,7] Several other types of dielectric resonators such as the dielectric circular ring resonator[7] and rectangular resonators[8] have also been studied. The sphere is used here as an example to validate some of the statements made in the section on Theory. An attempt will be made to show principally that the energy associated with the dielectric resonance outside the sphere can be neglected compared to the energy stored inside the sphere, and that the skin losses can be neglected compared to the dielectric losses.

Richtmyer[7] has shown in general that energy cannot be confined in a finite region of space by a dielectric alone because energy is always lost by radiation. Under certain conditions, though, this energy loss can be quite small compared to the dielectric losses. The electromagnetic fields outside the dielectric sample initially decay rapidly but then at large distances from the sample they oscillate. Because of the presence of this radiation field, the integral over the total energy density outside the dielectric resonator does not converge. A related problem with dielectric resonators is how much of the energy outside the sample should be included in the stored energy of the resonator.

Because of these problems we will consider the dielectric resonator in a slightly different way. Experimentally we know the spherical dielectric sample is not radiating since it is surrounded by waveguide. We will consider the problem of a dielectric sphere of radius $a$ and relative dielectric constant $\epsilon$ at the center of a spherical metal cavity of radius $b>a$. For estimating the skin losses, $b$ will be some number the order of the waveguide dimensions. We will consider the modes which are little different from the modes which would exist if the sample were in free space.

The electromagnetic fields can be expressed in terms of the vector potential in spherical coordinates, which inside and outside the dielectric sphere will be

$$\boldsymbol{A}_{\text{in}} = j_n(k_1 r) e^{im\phi} \left[ \hat{\theta} \frac{im}{\sin\theta} P_n{}^m - \hat{\phi} \frac{dP_n{}^m}{d\theta} \right] e^{-i\omega t},$$

$$\boldsymbol{A}_{\text{out}} = [B_n j_n(k_2 r) + C_n n_n(k_2 r)] e^{im\phi} \cdot \left[ \hat{\theta} \frac{im}{\sin\theta} P_n{}^m - \hat{\phi} \frac{dP_n{}^m}{d\theta} \right] e^{-i\omega t}.$$

$j_n$ and $n_n$ are spherical Bessel functions of the first and second kind, $P_n{}^m$ are the associated Legendre functions,[9]

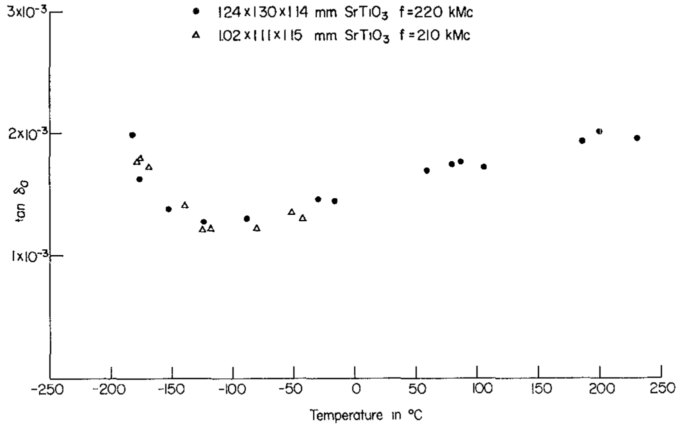


Fig. 3—Loss tangent vs temperature.

[6] P. Debye, "Der Lichtdruck auf Kugeln von Beliebigem Material," *Ann. Phys.*, ser. 4, vol. 30, pp. 57–136; 1909.

[7] R. D. Richtmyer, "Dielectric resonators," *J. Appl. Phys.*, vol. 10, pp. 391–398; June, 1939.

[8] N. M. Kroll, *et al.*, "Millimeter Wave Measurements," Columbia University, New York, N. Y., Radiation Lab. Quart. Rept., June 16 to September 15, 1959.

[9] For the properties of spherical Bessel function and associated Legendre functions, see J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., Inc., New York, N. Y., 1941, or a similar work.

$k_1 = (2\pi/\lambda)\sqrt{\epsilon}$, $k_2 = 2\pi/\lambda$, and $\omega$ is the angular frequency. $B_n$ and $C_n$ are determined by the boundary condition that the tangential components of $\boldsymbol{A}$ and its curl must be continuous at $r=a$, and that the tangential components of $\boldsymbol{A}$ must vanish at $r=b$. A characteristic equation is obtained which determines the resonant frequencies of the system.

$$\frac{[k_1 a j_n(k_1 a)]'}{j_n(k_1 a)} = \frac{[(k_2 a n_n(k_2 a)]' - \dfrac{n_n(k_2 b)}{j_n(k_2 b)}[k_2 a j_n(k_2 a)]'}{n_n(k_2 a) - \dfrac{n_n(k_2 b)}{j_n(k_2 b)} j_n(k_2 a)}.$$

The prime indicates differentiation with respect to the argument. The damping because of skin losses can be computed and expressed in terms of a $Q_{\text{skin}}$.[10]

$$Q_{\text{skin}} = \frac{b}{\delta} + \frac{a}{\delta}(\epsilon - 1)(k_2 a)^2 (k_2 b)^2 \cdot \{ n_n(k_2 a) j_n(k_2 b) - j_n(k_2 a) n_n(k_2 b)\}^2,$$

where $\delta$ is the skin depth $(2/\omega\mu\sigma)^{1/2}$ and $\sigma$ is the conductivity of the walls. We have assumed that the relative permeability of the cavity and of the dielectric sphere is one.

The radius of the samples measured at 20 kMc was of the order of $5 \times 10^{-4}$ meters. If we assume that $b = 15 \times 10^{-4}$ meters, which is roughly $\frac{1}{2}$ the smallest dimension of the waveguide, we find that $\epsilon = 150$ and $\epsilon = 750$ for the first two roots of the characteristic equation for $n=1$. The characteristic equation is almost independent of $b$ for values of $a$, $b$, and $\omega$ of this order. If we assume that the metal cavity is copper ($\sigma \approx 5.8 \times 10^{-7}$ mhos/meter), we find $Q_{\text{skin}} \approx 10^5$ and $Q_{\text{skin}} \approx 5 \times 10^5$ for $\epsilon = 150$ and 750, respectively. For larger values of $n$, $Q_{\text{skin}}$ is still larger. Since the loss tangent of the material being measured was the order of $10^{-3}$ which corresponds to an unloaded $Q = 10^3$, the skin losses may be neglected.

The ratio of the energy stored inside the dielectric sphere to the energy stored outside the dielectric sphere can be calculated. Considering roots of the characteristic equation such that the electric field decreases outside the dielectric sphere, it is found that for any reasonably small value of $b$ the energy stored inside the dielectric sphere is much greater than the energy stored outside. For instances for $n=1$, $\epsilon = 300$, $a = 5 \times 10^{-4}$ meters, and a frequency of 20 kMc, we find that $b$ must be about 0.8 meter before the energy stored externally equals the energy stored inside the dielectric sphere.

If we assume that $a = 5 \times 10^{-4}$ meters and $b = 15 \times 10^{-4}$ meters and $n=1$, then the ratio of the energy stored outside the dielectric sphere to the energy stored inside is 0.03 for $\epsilon = 150$ and 0.007 for $\epsilon = 750$. For larger values of $n$ and larger values of the dielectric constant, the ratio of the energies is still smaller.

Other shapes of dielectric resonators such as cubes or parallelepipeds will behave in a similar manner in regard to skin losses and the relative energy stored inside and outside the sample; although the details will depend on the specific geometry. Some of the possible uses of dielectric resonators have been pointed out by Okaya.[11]

[10] W. R. Smythe, "Static and Dynamic Electricity," McGraw-Hill Book Co., Inc., New York, N. Y., p. 531; 1950.

[11] A. Okaya, "The rutile microwave resonator," Proc. IRE, (Correspondence), vol. 48, p. 1921; November, 1960.